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# FREE ELECTRON LASER PROJECT



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ELECTROSTATIC ACCELERATOR FREE ELECTRON LASERS\*

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\*Research supported in part by the U.S. Office of Naval Research Contract No. N000-14-20-C-0308 1/81.

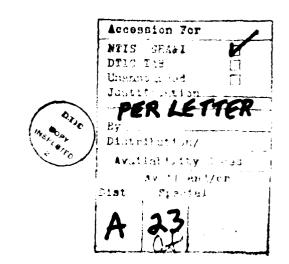
OctoBER 28, 1981



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#### INTRODUCTION.

The amplification of short wavelength coherent electromagnetic radiation by relativistic electrons moving through a spatially periodic transverse magnetic field was first demonstrated at Stanford University [4]. These experiments were carried out using the bunched electron beam emerging from a radio frequency linear accelerator. Although the electron beam quality was ideally suited to study the most important operating characteristics of the free electron laser, the small amount of available average electron beam current coupled with only a small laser extraction efficiency contributed to limit both the amount of average laser power produced (P=0.5 watts) and the overall operating efficiency of the device (e<0.1%).

Since the Stanford experiments a considerable amount of work has been done to study various schemes directed toward the development of efficient high power free electron lasers. In some of the schemes high single pass laser extraction efficiency is pursued using for example variable parameter wigglers [2], constant period wigglers consisting of only a few magnet periods [3] and constant period gain-expanded wigglers [4]. In other schemes the electron beam is recirculated several times through the laser interaction region [4,5] to increase total overall efficiency while retaining the characteristically small single pass efficiency of a constant period wiggler.

The present paper addresses the problem of increasing the power and efficiency of free electron lasers from a point of view which is fundamentally different from the schemes mentioned above. The

schemes discussed here are based on the utilization of the continuous electron beams generated by electrostatic accelerators. The basic idea is to recover the energy and charge of the electron beam after it has interacted with the free electron laser. This scheme was first suggested by Madey [6] in 1970 and later pursued by Elias [7] in 1978 to develop the two-stage FEL concept.

As will be discussed in detail in the next section two major changes occur as a result of recovering the energy and charge of the electron beam produced by electrostatic accelerators. First, the everage amount of electron beam current that can be extracted from the high voltage terminal, at constant voltage, can be increased from typical values of a few hundred microamperes to several amperes of average beam current. Second, even with a low single pass FEL energy extraction efficiency the overall efficiency of the device can be potentially very high because the energy losses occurring during the electron beam recovery stage can be made substantially smaller than the amount of laser energy produced. It will thus be shown here that as a result of electron beam recovery a considerable amount of average laser power (P>10 kW) can be generated with high everall laser efficiency (e>50%) using electrostatic accelerators. In addition to high current operation, electrostatic accelerators are well suited to provide the excellent quality electron beams demanded by free electron lasers. It is also worth noting that the operation of a free electron laser with electron beam recovery reduces substantially the amount of ionizing radiation normally produced when the spent electron beam is suddenly stopped. This feature alone may be of primary importance to those considering using free electron lasers for commercial or laboratory applications.

Three schemes will be discussed here: a) short pulse operation with no energy recovery, b) CD single-stage operation with energy recovery and c) CW two-stage operation with energy recovery. Also, a review is made of the electron beam quality required by the FEL.

## ELECTROSTATIC ACCELERATOR FEL WITH I'D ENERGY RECOVERY.

The technology of high-voltage electrostatic ac celerators is now well established. Since the 1950s these machines have been operated quite reliably to produce very high quality continuous beams of electrons or ions in the radium voltage range from 1 MV to 25MV. The maximum BC beam current ( $I_B$ ) that can be extracted during conventional operation from these devices is entirely determined by the maximum amount of charging current ( $I_C$ ) required to maintain the LV terminal charged at constant electric potential. Extracting more heam current than the charging current ( $I_B > I_C$ ) results in a situation whereby the electric potential of the high-voltage terminal and hence the electron's kinetic energy will decrease steadily with

time. During normal operation (no energy recovery) these devices are capable of generating on a steady state basis from a few tens of milliamps of beam current at low voltage to a few hundred microamperes at high voltage. A schematic diagram of a single stage free electron laser using an electrostatic accelerator without electron beam recovery is shown in Figure 1.

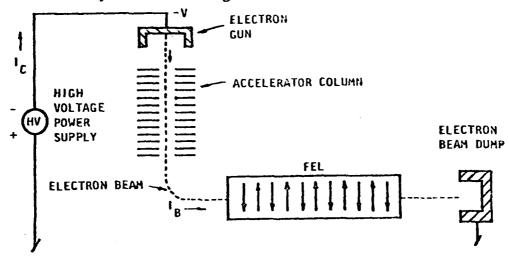


Figure 1. An electrostatic accelerator FEL operating with no beam recovery.

The high-voltage terminal is charged to a potential -V by means of an electrically charged moving belt or pelletron chain. An electron gun located in the HV terminal produces a relatively low voltage electron beam which is subsequently injected into and then accelerated to its final energy by the constant electric field of the accelerating column shown. After interacting with the FEL, the electron beam is stopped at the electron beam dump. There, most of the beam's energy is converted to heat and ionizing radiation. Hence, the overall efficiency of the laser is low since only a small amount of energy is actually converted into laser radiation. It was noted earlier that it is possible to increase the single pass efficiency of the laser by means of variable parameter wigglers. However, the amount of average laser power obtained in this configuration is still small due to the limited amount of average current available from the accelerator in this mode of operation.

Assuming that I<sub>C</sub> is the charging current reaching the high voltage terminal and I<sub>B</sub> is the electron beam current extracted from the accelerator, then the rate of change of voltage with time can be readily calculated as follows

$$\frac{dV}{dt} = -\frac{\left[I_B - I_C\right]}{C} \tag{1}$$

where C is the electrical capacitance to ground of the high-voltage

terminal. Typically C=200 picofarad. For constant wavelength operations, the free electron laser operating in the single particle regime requires an electron beam whose energy spread is smaller than the energy width of the gain curve. This requirement imposes a maximum acceptable drop in the HV electrostatic potential of

there N is the number of FEL wiggler periods. Equation (1) and (2) can be combined to yield a value for the maximum electron pulse length that can be used with a free electron laser operating in this mode:

$$[\Delta t]_{MAX} = \frac{cv}{2N(I_B^{-1}c)}$$
 (3)

Before another electron pulse can be initiated, the accelerator HV terminal value must be recharged to its initial potential. The charging rate is given by

$$\frac{dV}{dt} = \frac{I_C}{C} \tag{4}$$

The total recharging time can thus be calculated combining equations (2) and (4) to obtain

$$[\Delta t]_{CH} = CV/2NI_{C}$$

It follows that the maximum pulse repetition rate that can be obtained in this mode of operation is:

$$PRR = \frac{1}{[\Delta t]_{MAX} + [\Delta t]_{CH}}$$
 (5)

Table 1 below summarizes typical operating characteristic of an electrostatic accelerator FEL when no electron beam energy recovery techniques are used. The results shown were obtained using the above equations with C = 200 picofarad and N = 250.

Table 1. Performance of an electrostatic accelerator free electron laser using no electron beam recovery techniques.

٧	I <sub>B</sub>	1 c	Р	P	[At] <sub>MAX</sub>	PRR	
5MV	2A	500µA	20kW	5W	10 <sup>-6</sup> sec	250hZ	
5MV	100A	500µA	1MW	5W	20x10 <sup>-9</sup> sec	250Hz	

Clearly a free electron laser operating in the above described configuration can be useful in many laboratory and commercial applications where laser power and efficiency is not of primary consideration and where sufficient protection exists against ionizing

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radiation produced at the electron beam dump. However, if higher power and overall efficiency is required, then the electron beam energy and charge must be recovered. An appropriate technique to achieve this is discussed in the next section.

#### ELECTROSTATIC ACCELERATOR FEL WITH ENERGY RECOVERY.

As noted in the introduction, the power and efficiency of a free electron laser can be substantially improved if the energy of the spent electron beam is recovered. Using electrostatic accelerators this is done in a straight forward way as shown in Figure 2.

After interacting with the FEL the spent electron beam's kinetic energy is reduced from a few megavolts to a few kilovolts by the electrostatic decelerating column shown in the figure. Subsequently, the relatively low kinetic energy beam enters the electron charge collector where the electrons are separated according to energy and captured by the collector surfaces with minimum production of heat or ionizing radiation. The technique of recovering electron beam energy by means of "depressed collectors" is used frequently with many modern microwave tubes as discussed by Nechtel [8].

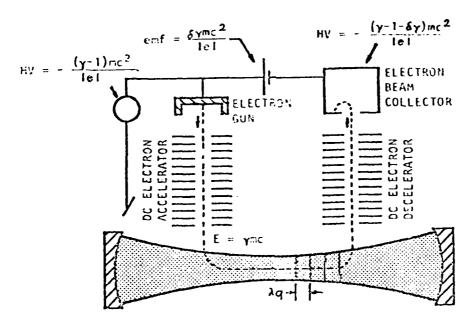


Figure 2. An electrostatic accelerator FEL operating with electron beam energy recovery.

The battery shown between the cathode and collector replaces the energy lost by the electron beam to FEL radiation.  ${\bf 1}_R$ 

represents the amount of electron beam current recovered. Note that  $\mathbf{I}_R$  is a conduction current while  $\mathbf{I}_B$  is a true beam current. Equation (1) must be modified to include the recovered current  $\mathbf{I}_R$ 

$$\frac{dV}{dt} = -\frac{1}{C} [I_B - I_C - I_R]$$
 (6)

It follows from the above equation that a steady state regime can be obtained  $(\frac{dV}{dt} = 0)$  when

$$I_{B}^{-1}I_{R} = I_{C} \tag{7}$$

That is to say, the potential of the high-voltage terminal will not change if the amount of electron current  $(1_B-I_R)$  lost in the system is equal to the charging current  $I_C$ . For this scheme to work it is thus important to recover as much of the electron beam current as it is possible. However, even if all of the beam current is not collected it is still possible to operate the FEL with reasonably large values of power and efficiency. For example, assume that  $I_D=I_R(1-\alpha)$  where is the fraction of beam current lost. The maximum electron pulse length that can be used with a FEL is obtained by modifying equation (3) to read:

$$[\Delta t]_{MAX}^{R} = \frac{CV}{2N(I_{B}^{-1}I_{C}^{-1})} = \frac{CV}{2N(\alpha I_{B}^{-1}I_{C}^{-1})}$$
(8)

For example, if 10% of the initial beam current cannot be recovered then using (6) with  $\alpha=0.1$  and the values for C,V,N,IB, and IC used in the example discussed in the previous section the following result is obtained:

$$[\Lambda t]_{MAX}^{R} = \frac{[\Lambda t]}{\alpha} = 10[\Lambda t]_{MAX}$$

Hence, the maximum pulse length that can be used with the FEL has increased from  $[\Delta t]_{MAX}$  with no energy recovery to 10 times  $[\Delta t]_{MAX}$  when 10% of the beam current is not recovered. Also, in this example the average power and overall efficiency has also been increased by a factor of 10. The ideal situation is, of course, to recover all of the electron beam current.

Table II summarizes the possible performance of single-stage electrostatic accelerator free electron lasers having various levels of electron beam energy and current recovery. The efficiency figure is defines as follows:

$$e = \frac{AVERAGE POWER}{PEAK POWER} \times 100\%$$
.

Table II. Performance of electrostatic accelerator free electron lasers with various degrees of energy recovery. (V=5MV, C=200 pi-cofarads,  $I_R$ =2A,  $I_C$ =500  $\mu$ A, N=250)

3	P(peak)	P	[it] <sub>MAX</sub>	Efficiency	
1	20kW	5W	10 <sup>-6</sup> sec	.025%	
0.1	20kW	50W	10 <sup>-5</sup> sec	.025%	
0	20kW	20kW	တ	100%	

The results shown in Table II indicate that with electron beam energy recovery ( $\alpha \le 1$ ) it is possible to operate FELS at high power and high overall efficiency using electrostatic accelerators even if the charging current  $I_C$  is small. Also, since during the electron beam collection process the electrons have only small kinetic energies, the amount of ionizing reaction produced is small. A more detailed discussion of the electron collection process can be found elsewhere in this book under the title "The UCSB FEL Experimental Program".

Note that in the calculation of overall efficiency, power supply losses have not been included. If these losses are taken into account, then in some cases the overall laser efficiencies are expected to be as high as 50% if all the charge and energy in the electron beam is recovered.

#### TWO-STAGE FREE ELECTRON LASERS USING ELECTROSTATIC ACCELERATORS.

The two-stage FEL concept [3] was developed in 1978 at Stanford University as a means of generating tunable coherent radiation at short wavelengths using low energy electron beams, such as the ones available from electrostatic accelerators. If the techniques of electron beam energy recovery discussed previously are also used with two-stage FELs then a considerable amount of laser power can be produced in the 1000Å to 50 µm wavelength range using conventional low voltage electrostatic accelerator. The simplest configuration of a two-stage FEL is illustrated in Figure 3.

As shown in the figure a continuous beam of monochromatic electrons of energy Emy mo<sup>2</sup> emerges from the electrostatic accelerator column shown on the left side of the figure. The beam interacts with the FEL wiggler to excite a long vavelength laser TEMoo mode which resonates between the two spherical mirrors. The wavelength of this mode is given approximately by the relation

$$\lambda p = \frac{\lambda o}{2x^2}$$

there  $\lambda$  is the period of the magnetic wiggler,  $\lambda_{mc}^2$  is the energy of the incoming electrons. The resonator mirrors are constructed of highly reflective materials at the operating wavelength  $\lambda_p$  to allow the intensity of the optical mode to grow to values in the range  $10^{4}10^{9} \rm Mwatts/cm^2$ . At this high level of optical power density

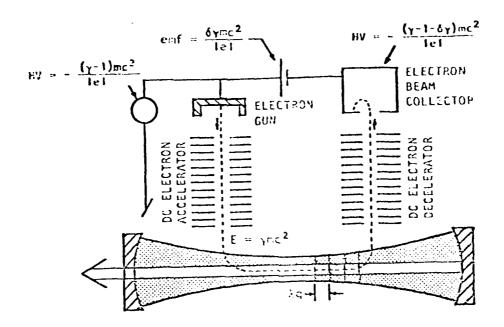


Figure 3. A simple two-stage FEL scheme using electrostatic accelerators with electron beam energy recovery.

the same electron beam can interact again with the intense optical mode to produce coherent radiation at a much shorter wavelength

$$\lambda = \frac{\lambda p}{4\gamma^2} = \frac{\lambda o}{8\gamma'}$$

The short wavelength optical mode (second-stage FEL) is shown in white as a TEMoo gaussian mode propagating along the axis of the resonator.

A second two-stage FEL scheme is illustrated in Figure 4. Here, separate electron beams are used to excite independently the first and second FEL stages. The major advantages of this scheme are: a) the wavelength of the second stage can be tuned without perturbing the operation of the first stage, b) the small signal gain of the second stage laser can be optimized by choosing correctly the ratio

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of pump wavelength  $\lambda$  to second-stage wavelength  $\lambda$  and c) the FEL interaction length of the second stage can be adequately controlled using independent electron beam optic components. Table IV summarizes the operating characteristics of a two-stage FEL operating at two different wavelengths.

### ELECTRON BEAM REQUIREMENTS.

A. Beam Quality Requirements. In a free electron laser the axial velocity  $\boldsymbol{\beta}_Z$  of the electron beam determines whether or not the electrons radiate coherently. The maximum spread of axial velocities that can be accepted by a constant period FEL wiggler can be calculated from the energy width of the FEL gain curve at fixed wavelength. The maximum velocity spread that can be accepted by a FEL is given by

$$[\delta \beta_z]_{MAX} = \frac{1}{2i!\gamma^2} \tag{9}$$

where N is the number of magnetic periods in the wiggler and  $\gamma mc^2$  is the relativistic energy of the electron beam. If  $\beta$  is the total speed of an electron in the beam and  $\beta_i$  is its total transverse speed then in a FEL:

$$\beta^{2} = \beta_{z}^{2} + \beta_{i}^{2} = \beta_{z}^{2} + \frac{K^{2}}{\gamma^{2}} + \beta_{io}^{2}$$
 (10)

there  $\gamma = \frac{1e18\lambda_0}{2umc\gamma}$  is the transverse speed (MKS units) acquired by the electron from the magnetic wiggler. B is the rms value of the magnetic field on axis and  $\lambda_0$  is the periodicity of the magnetic wiggler structure. 3 is the transverse drift velocity of the electrons with respect to the axis of the wiggler.  $\beta_0$  is finite if the electron is injected into the magnetic structure at the wrong angle. Changes in  $\beta_0$  can thus originate from variations in B and  $\beta_0$ . Equation (10) can be used to estimate separately the contribution to  $[\delta\beta_0]$  from variations in B and  $\beta_0$ . If  $\beta$  is held fixed then a) for fixed  $\beta_0$ .

$$\left[\delta \varepsilon_{z}\right]_{B} = -\frac{K^{2} \delta S}{2\gamma^{2}\beta_{z}B} \tag{11}$$

and 2) for fixed B  $\left[ \hat{\beta}_{2} \right]_{B_{10}} = -\frac{\beta_{10} \hat{\beta}_{10}}{\beta_{2}} - \frac{\beta^{2}}{\beta_{2}}$  (12)

where it has been assumed that 68  $_{10}$   $_{10}$  . In a constant period magnetic the magnetic field at a district x from the exist is given approximately by:  $B(x,z) = B_0 \cosh(\frac{2\pi x}{30}, \cos(\frac{2\pi x}{30}))$ , for small x,  $\delta B/B = \frac{2\pi x}{30}$  is the fractional change in magnetic field near the exist of the wiggler. Using this relation and equations (9) and (10) it is possible to estimate the maximum electron beam radius F that can be used with a given FTE wiggler.

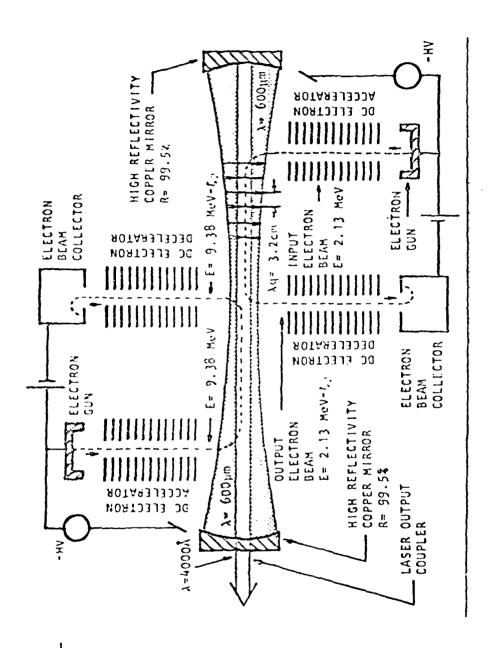


Figure 4. A twostage FEL using two independent electron beams.

$$R = \frac{\lambda o}{2\pi K} \sqrt{\frac{\beta_z}{N}}$$
 (13)

From equations (12) and (9) a maximum acceptable electron beam divergence angle can be derived:

$$\theta = \frac{1}{\gamma \sqrt{2N_3^3}} \tag{14}$$

And finally from relations (13) and (14) a maximum acceptable transverse electron beam emittance can be obtained:

$$\varepsilon \leq \theta R = \frac{\lambda_0}{2\pi \sqrt{2N_Y K}} \tag{15}$$

The transverse effects introduced through equation (11) can in principle be minimized provided low wiggler magnetic fields are used. This is, of course, accomplished at the expense of reducing the available small signal optical gain. Also, reducing the transverse dimensions of the electron beam will result in a smaller value of [53] in equation (11). Equation (12) describes the contribution to  $\delta_3$  resulting from the spread in transverse electron velocities in the beam. The sources of such transverse electron velocities can be traced back to the electron gun cathode (thermal effects) and/or to the electron beam accelerator. Their contributions to  $[\delta_3]$  can be described as follows:

$$\frac{\left[\delta\beta_{z}\right]_{T_{II}}}{\left[\delta\beta_{z}\right]_{MAX}} = \frac{\frac{1}{\gamma^{3}}\sqrt{\frac{K\overline{1}}{mc^{2}}}}{\frac{1}{2N}} = \frac{\left(2N\right)\sqrt{\frac{K\overline{1}}{mc^{2}}}}{\left(16\right)}$$

$$\frac{\left[\delta\beta_{z}\right]_{T_{l}}}{\left[\delta\beta_{z}\right]_{MAX}} = \frac{\frac{1}{\gamma_{4}}\sqrt{\frac{KT}{mc^{2}}}}{\frac{1}{2N\gamma^{2}}} = \left(\frac{2N}{\gamma^{2}}\right)\left(\frac{KT}{mc^{2}}\right) \tag{17}$$

$$\frac{\left[\delta\beta_{z}\right]_{\text{EMITTANCE}}}{\left[\delta\beta_{z}\right]_{\text{MAX}}} = \frac{3 \times 10^{-5} \frac{J}{\gamma^{2} \hat{\beta}^{2}}}{\frac{1}{2N\gamma^{2}}} = \frac{6 \times 10^{-5} \text{N J}}{\beta^{2}}$$
(18)

where T is the cathode temperature,  $[6\beta_Z]_{T_0}^T$  and  $[\delta\beta_Z]_{T_0}^T$  are the contributions to  ${}^{6\beta}_Z$  from variations in longitudinal and transverse cathode thermal velocities respectively.  $[{}^{6\beta}_Z]_{EMITTANCE}^T$  is an empirical relation between transverse emittance e(mm-nred) and electron beam current I(KA) derived by Lawson and Penner[9]:

$$e = \frac{0.3\sqrt{1}}{\gamma\beta}$$

Table 3 lists the electron beam quality requirements necessary to operate single-stage and two-stage IMAs.

JMIN is the minimum current density required to operate the FEL with a 10% small signal gain/pans. JMAX has been calculated from the Lawson-Penner conditions and:

$$\frac{\left[3\beta_{z}\right]_{EMITTANCE}}{\left[3\beta_{z}\right]_{MAX}} = 1$$

$$J_{MAX} = \frac{10^5}{6N} \left( \frac{Amp}{cm^2} \right)$$

Table 3. Electron Beam Quality Calculations

	SINGLE STAGE FEL		TWO-S	TWO-STAGE FEL	
λ Υ R(mm) N KT(eV)	100pm 7 2 200 0,2	4μm 50 2 200 0.2	16µm 7 0.4 600 0.2	4000Å 20 1.3 8000 0.2	
J <sub>MIN</sub> ( <del>Amp</del> ) (10% gain)	3	10	100	5000	
$\frac{\left[\delta\beta_{z}\right]^{T_{H}}}{\left[\delta\beta_{z}\right]_{MAX}}$	<b>0</b> .62	0.003	0.06	0.28	
[68 <sub>z</sub> ] <sub>MAX</sub>	10 <sup>-5</sup>	10 <sup>-6</sup>	3×10 <sup>-5</sup>	10 <sup>-4</sup>	
J <sub>MAX</sub> (Gmp/cm <sup>2</sup> ) (from Lawson- Penner eq.)	100	100	33	3	

J<sub>MAX</sub> depends only on the number of wiggler periods. The calculations listed in Figure 5 indicate that the major source of originates from the emittance relations Lawson and Penner. It can be seen from Table 5 that to operate two-stage FELs at short wavelength (=4000A) the transverse emittance of electron beams has to improve by a factor of 100 with respect to the value calculated from the Lawson-Penner relation. Single-stage FELs, on the other hand, can operate with the emittance calculated from the Lawson-Penner relation.

b. Bear Current Requirements and Laser Power Output. The optical stall signal particle region can be written in KMS units as follows

$$G = \frac{5.24 \times \frac{37^2}{2}}{(1+K^2)^3} \frac{57}{7} \frac{B^2 + N^3}{r^2}$$

#### where:

 $\lambda$  = signal wavelength

λo= magnet period

B = RMS magnetic field on axis

I = electron beam current

N = number of magnet periods

r = optical beam radius

K = g A o B/2 mmc

The above gain equation has been normalized to give the correct gain value for the Stanford FEL. It is assumed here that the electron beam radius R is smaller or equal to the optical beam radius. At saturation (i.e. when the small signal gain is reduced by a factor of 2) the amount of power that can be extracted from the electron beam as laser radiation is

$$\overline{P} = \frac{IV}{2N}$$

The electron beam requirements and the typical expected performance of a single-stage free electron laser has been incorporated into Table 4. Similarly Table 5 summarizes the operating characteristics of a two-stage FEL based on the selected illustrated in Figure 4.

Table 4. Performance of a single-stage FEL and required electron beam characteristics.

Wavelength (µm)	360
Magnet period (cm)	3
# magnet periods	100
Magnetic field (r)	0.06
Small signal gain (Amp <sup>-1</sup> )	0.60
Average laser power (kW/Amp)	15
Overall efficiency (%)	-50
Elect. beam energy (MeV)	3
Maximum transverse emittance (paramal)	n122
Kaximum Aδ/γ	5×10 <sup>-3</sup>

Table 5. Performance of a two-stage FEL and required electron beam characteristics.

Wavelength (µm)	0.4	16
Pump wavelength (μm)	600	4000
Pumpwave intensity (MW/cm <sup>2</sup> )	<b>2</b> 50	60
Interaction length (m)	2.4	1.2
Small signal gain $(Amp^{-1})$	5x10 <sup>-3</sup>	8×10 <sup>-3</sup>
Power output (kW/Amp)	0.5	2
Overall efficiency (%)	0.3	1.5
Elect. beam energy (MeV)	9.38	<b>3.</b> 55
Maximum transverse emittance (mm-mrad)	π0.8	π10
Maximum Ay/y	6×10 <sup>-5</sup>	$0.8 \times 10^{-3}$

#### CONCLUSIONS.

The operations of single-stage and two-stage free electron lasers using the electron beams produced by electrostatic accelerators has been discussed. The techniques of electron beam energy recovery reviewed in this chapter can be used to produce intense beams of coherent electromagnetic radiation in the far infrared region with high levels of efficiency with present electrostatic accelerator technologies. At shorter wavelength high power laser radiation can also be produced, but only at reduced overall efficiency. It may be possible be produced, but only at reduced overall efficiency. It may be possible to extend the operating wavelength region of single-stage electrostatic accelerator free electron lasers into the visible region with the advent of new HV technology.

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